

Exercise 30

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = x^{3/2}$$

Solution

The domain of $f(x) = x^{3/2} = \sqrt{x^3}$ is

$$x^3 \geq 0$$

$$x \geq 0$$

$$\{x \mid x \geq 0\}.$$

Calculate the derivative of $f(x)$ using the definition.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} \cdot \frac{(x+h)^{3/2} + x^{3/2}}{(x+h)^{3/2} + x^{3/2}} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h[(x+h)^{3/2} + x^{3/2}]} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 3xh + h^2}{(x+h)^{3/2} + x^{3/2}} \\ &= \frac{3x^2}{x^{3/2} + x^{3/2}} \\ &= \frac{3x^2}{2x^{3/2}} \\ &= \frac{3}{2}x^{1/2} \end{aligned}$$

The domain of $f'(x) = \frac{3}{2}\sqrt{x}$ is

$$x \geq 0$$

$$\{x \mid x \geq 0\}.$$